

ERRATUM TO “SPHERICAL CLASSES AND THE ALGEBRAIC TRANSFER”

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Let Q_0S^0 be the basepoint component of $QS^0 = \lim_n \Omega^n S^n$. A spherical class in Q_0S^0 is an element belonging to the image of the Hurewicz homomorphism:

$$H : \pi_*^s(S^0) \cong \pi_*(Q_0S^0) \rightarrow H_*(Q_0S^0).$$

Here and throughout this note, homology is taken with coefficients in \mathbb{F}_2 , the field of two elements. The long-standing conjecture on spherical classes reads as follows.

Conjecture 1.1. There are no spherical classes in Q_0S^0 , except the elements of Hopf invariant 1 and those of Kervaire invariant 1.

An algebraic version of this problem goes as follows. Let V_k be a k -dimensional vector space over \mathbb{F}_2 . Then, the polynomial algebra in k variables $P_k = H^*(BV_k)$ is a module over both the Steenrod algebra \mathcal{A} and the general linear group $GL_k = GL(k, \mathbb{F}_2)$. J. E. Lannes and S. Zarati constructed homomorphisms

$$\varphi_k : Ext_{\mathcal{A}}^{k, k+i}(\mathbb{F}_2, \mathbb{F}_2) \rightarrow (\mathbb{F}_2 \otimes_{\mathcal{A}} P_k^{GL_k})_i^*$$

(see [6]) and have shown that these maps correspond to an associated graded of the Hurewicz homomorphism. The proof of this assertion is unpublished, but it is sketched by J. E. Lannes [5] and by P. G. Goerss [1]. The Hopf invariant 1 and the Kervaire invariant 1 classes are respectively represented by certain permanent cycles in $Ext_{\mathcal{A}}^{1,*}(\mathbb{F}_2, \mathbb{F}_2)$ and $Ext_{\mathcal{A}}^{2,*}(\mathbb{F}_2, \mathbb{F}_2)$, on which φ_1 and φ_2 are non-zero. Therefore, we are led to the following conjecture.

Conjecture 1.2. $\varphi_k = 0$ in any positive stem i for $k > 2$.

In the introduction of the article [2] we are mistaken in asserting that Lannes and Zarati’s work shows that Conjecture 1.2 implies Conjecture 1.1. This comes from the usual problem with spectral sequences: if an element maps to an element of higher filtration, then in the associated graded it will map to 0. Thus φ_k could be 0 even if H is not. Of course, if Conjecture 1.2 were false on a permanent cycle, then Conjecture 1.1 would also be false.

Apart from this, all of our results and proofs in the article [2] are correct.

This correction also applies to our papers [4] (joint with F. P. Peterson) and [3].

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